1990

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when $t = 1$ is $x(1) = 3$.

   (a) Find the values of t for which the particle is at rest.

   (b) Write an expression for the position $x(t)$ of the particle at any time $t \geq 0$.

   (c) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

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2. Let f be the function given by $f(x) = \ln\left(\frac{x}{x-1}\right)$.

   (a) What is the domain of f?

   (b) Find the value of the derivative of f at $x = -1$.

   (c) Write an expression for $f^{-1}(x)$, where f^{-1} denotes the inverse function of f.

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3. Let R be the region enclosed by the graphs of $y = e^x$, $y = (x-1)^2$, and the line $x = 1$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

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4. The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second.

   (Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

   (a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

   (b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

   (c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

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5. Let f be the function defined by $f(x) = \sin^2 x - \sin x$ for $0 \le x \le \frac{3\pi}{2}$.

(a) Find the x-intercepts of the graph of f.

(b) Find the intervals on which f is increasing.

(c) Find the absolute maximum value and the absolute minimum value of f. Justify your answer.

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6. Let f be the function that is given by $f(x) = \frac{ax + b}{x^2 - c}$ and that has the following properties.

(i) The graph of f is symmetric with respect to the y-axis.

(ii) $\lim_{x \to 2^+} f(x) = +\infty$

(iii) $f'(1) = -2$

(a) Determine the values of a, b, and c.

(b) Write an equation for each vertical and each horizontal asymptote of the graph of f.

(c) Sketch the graph of f in the xy-plane provided below.

Note: The xy-plane is provided in the pink test booklet only.

END OF EXAMINATION.